BACKPAPER EXAM, ALGEBRA III, TOTAL MARKS: 100

All questions carry equal marks.

(1) Is $\mathbf{F}_3[x]/(x^2+x+1)$ a field? Is $\mathbf{F}_5[x]/(x^2+x+1)$ a field?
(2) Determine all ideals of $\mathbb{R}[[t]]$. Here $\mathbb{R}[[t]]$ is the ring of all formal power series (expressions of the form $\sum_{n=0}^{\infty} a_n T^n$) in one variable t with real coefficients a_n.
(3) Classify all rings of order 10.
(4) Classify all finite abelian groups of order 144.
(5) Determine the ring obtained from $\mathbb{R} \times \mathbb{R}$ by inverting the element $(2,0)$.
(6) Let α be a Gauss integer, assume α has no integer factor and $\bar{\alpha}\alpha$ is a square integer. Then show that α is a square in $\mathbb{Z}[i]$.
(7) Write the abelian group generated by two elements x and y, with the relation $3x+4y=0$, as a direct sum of cyclic groups.
(8) Classify finitely generated module over the ring $\mathbb{C}[\epsilon]$ where $\epsilon^2=0$.
(9) Find a direct sum of cyclic groups isomorphic to the abelian group presented by the matrix

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 2 \end{pmatrix}$$

(10) Classify up to similarity all 3×3 matrices A over the complex numbers satisfying $A^3 = I$.